Comment on "Extensions of the Falkner-Skan Similar Solutions to Flows with Surface Curvature"

B. S. Massey* and B. R. Clayton†
University College, London, England

It is disappointing and surprising to note that Murphy[1] repeats his previous mistake[2,3] of making an order-of-magnitude analysis *before* differentiating his equations of motion. The errors so introduced have already been pointed out by the present writers[4] in presenting what they believe to be a correct and more far-reaching investigation. A number of other algebraic errors have been taken over from Ref. 3 to Ref. 1, e.g., in Eqs. (28) and (29) of Ref. 1. Furthermore, Murphy's integrations are continued to $\eta = 10$ and to $\Omega = \pm 0.1$, i.e., to $\Omega\eta = Ky = \pm 1$, a value that is not negligible in comparison with 1 as he assumed in deriving his approximate equations. Thus, although his qualitative conclusions about the effect of surface curvature are correct, all of Murphy's detailed results, except those for zero curvature, are at fault.

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* Senior Lecturer, Mechanical Engineering Department.

† Doctoral Student; now Research Engineer, Geoffrey Woods Ltd., Colchester, England.

Comments on "Transport Properties of Hydrogen"

R. S. Devoto*
Stanford University, Stanford, Calif.

In a recent paper, Brezing[1] has presented the results of his calculations of the transport properties of ionized hydrogen. In the light of research on transport properties of ionized gases performed here[2,3] and elsewhere, several statements and conclusions in his article are open to question. In order of their appearance they are as follows:

1) It has been proved that charge transfer affects only the $Q^{(l)}$ cross sections for odd l.[4] Hence, Eq. (9) for $Q^{(1)}$ is correct but $Q^{(2)}$ is not the same and must be computed taking into account only the elastic forces. The difference between these two cross sections can then be quite large. For argon $Q^{(1)}$ is typically $3 \times Q^{(2)}$.[3]

2) The thermal conductivity of binary mixtures is, in general, not proportional to the viscosity (remark just above Eq. (13) and also in Sec. III under thermal conductivity). In the special case of the fully ionized gas where the viscosity is effectively that of the ions, the thermal flux is carried mainly by the electrons, and the only interaction is Coulombic, then these two coefficients will be proportional. But they are not necessarily proportional over a large range of partial ionization where different intermolecular forces determine the two coefficients, i.e., atom-ion and atom-atom forces for the viscosity and electron-ion, electron-atom and electron-electron forces for the thermal conductivity.

3) Equation (18) is incorrect. From the correct form[5] we see that Eq. (18) is really the expression for the first approximation to D_{ei} ($\neq D_{ie}$) when atom and ion masses are equal, and that a compensating error in Eq. (19) results in the correct first approximation to the electrical conductivity. In the same section it is also implied that the second approximations to the multicomponent diffusion coefficients in the ternary mixture may be obtained by substituting the second approximation to the binary coefficients into Eq. (18) (after correction). Although this is probably true at very high or very low ionization, it has apparently not been proved in the intermediate case. The correct form of the second approximation to this coefficient has been given elsewhere.[2,6]

4) The most serious discrepancy occurs with the thermal conductivity λ. Brezing reports that the λ which he computed with the expressions of Ref. 5 differed by not more than 12% from the value obtained with the expression of Spitzer and Härm.[7] This conclusion disagrees with results from several sources. In what appears to be the earliest treatment of the properties of fully ionized gases, Landshoff[8,9] found that accurate values of the thermal conductivity could be obtained only if three or more terms were retained in the polynomial expansion of the perturbation to the electron equilibrium distribution function.† If two terms were used, as is done in Ref. 5, then the computed thermal conductivity is only 0.42 of the value obtained by Spitzer and Härm. Although Landshoff neglected the contribution of the ion in his treatment, its inclusion for hydrogen increases the thermal conductivity by only a few percent.[2] It is evident that values of the thermal conductivity as computed with the expressions of Ref. 5 must be viewed with scepticism when ionization is more than slight.

If there are no numerical errors in the computations of the thermal conductivity, there are two possible reasons for the apparent agreement of λ with that for the fully ionized gas. One possibility, which has occurred in other investigations, is that Brezing does not report the true thermal conductivity. This coefficient is evaluated correctly only when all of the diffusion fluxes in the mixture have been set equal to zero. (See Ref. 5, p. 483.) In this way, one is lead to the distinction between λ and λ' in Ref. 5. In the un-ionized gas λ' is within a few percent of λ so this distinction is not important. In fully ionized hydrogen, $\lambda' = 1.87\lambda$ in the second approximation,[2] so the distinction is very important. It does appear, however, that Brezing has taken this point into account.

Another possible source of the discrepancy is the Coulomb cross section. Several investigators have adjusted the numerical constants in this cross section to insure that the thermal conductivity takes on the correct value at full ionization. Eastlund[10] has inserted constants in the expressions of Ref. 5 to accomplish the same purpose. Since Brezing does not actually quote the expressions for either the charged cross sections or for the thermal conductivity, it is possible that he has followed one of these two approaches. Since neither has a theoretical basis, it leaves the resultant values of this coefficient open to question.

5) Discrepancies occur also in the values reported for the binary atom-ion diffusion coefficients. Brezing reports a

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* Research Associate, Department of Aeronautics and Astronautics. Associate Member AIAA.

† Note, however, that he calls the coefficients obtained with two terms the first approximation. This nomenclature differs from that of Chap. 7 of Ref. 5 and of Ref. 2.

maximum change of 18% from the first to second approximation to this coefficient. This large an increase seems unlikely since the ion-atom interaction (both charge transfer and elastic) is very much like a strong inverse-power potential for which the convergence is much faster. For argon, the third approximation to this coefficient is within 2% of the first over the temperature range from 5000°–20,000° K.[3]

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Comment on the Location of the Center of Pressure of a Right Circular Cone Using Newtonian Impact Theory

FRANK GARCIA JR.*
NASA Manned Spacecraft Center, Houston, Texas

TO obtain a proper estimate of the static pitching moment coefficient (C_m) for any body, it is necessary to have a knowledge of the location of the center of pressure, X_{cp}. Newtonian flow theory assumes that momentum horizontal to the surface remains unchanged, whereas that normal to the surface is completely absorbed at the surface. Theoretically, then, only the momentum normal to the surface will produce a force (dF) that is at an angle, η, to the freestream (Fig. 1). Using impact theory and looking at an element of area on a cone, this differential force acting on said element is

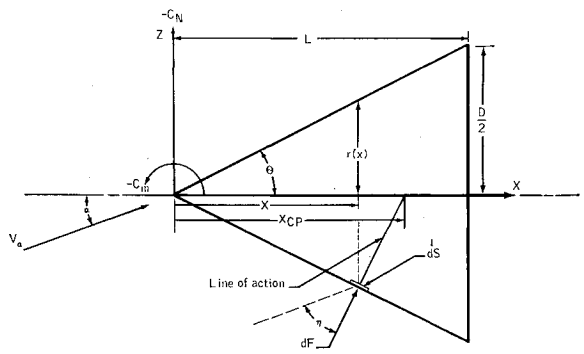

Fig. 1 Sketch of right circular cone.

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* Aerospace Technologist. Associate Member AIAA.

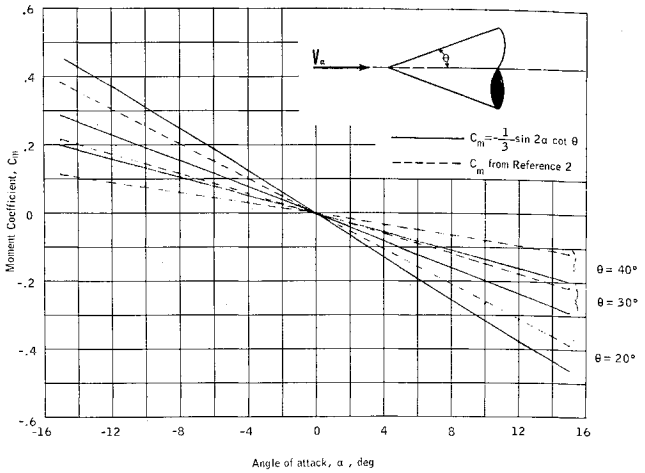

Fig. 2 Comparison of moment coefficients about the vertex for a right circular cone.

$$dF = qC_p dS$$

where

C_p = local pressure coefficient, $2\cos^2\eta$
q = dynamic pressure, $\frac{1}{2}\rho V^2$
dS = elemental surface area

The elemental force may now be translated along its line of action to the centerline of the cone and there resolved into normal and axial components, $-dC_N$ and dC_A, respectively. This translation will make the moment contribution of dC_A nil about points on the centerline of the cone. The differential moment about the vertex is then

$$dM = x_{cp}dN$$

where

$$dN = C_N'q(\pi D^2/4)d(x/D)$$

and from geometry of Fig. 1,

$$x_{cp} = f(x) = x\sec^2\theta$$

Reference 1 shows that when $\alpha \leq \theta$, the normal force coefficient per unit length may be expressed as

$$C_N' = -4(r/D)\sin2\alpha\sin2\theta = -8(x/D)\sin^2\theta\sin2\alpha$$

Making these substitutions in the moment equation and integrating over the length of the cone yields

$$M = -8q\pi\frac{D^3}{4}(\sin2\alpha\tan^2\theta)\int_0^{L/D}\left(\frac{x}{D}\right)^2 d\left(\frac{x}{D}\right)$$

$$M = -\tfrac{8}{3}q\pi(D^3/4)(\sin2\alpha\tan^2\theta)(L/D)^3$$

Likewise, performing the prescribed integration on dN, the following result is obtained.

$$N = -8\sin2\alpha\sin^2\theta q\pi\frac{D^2}{4}\int_0^{L/D}\left(\frac{x}{D}\right)d\left(\frac{x}{D}\right)$$

$$N = -4\sin2\alpha\sin^2\theta\, q\pi(D^2/4)(L/D)^2$$

If we now define the total center of pressure as

$$\bar{X}_{cp} = M/N$$